

Comments to the paper “Effect of Coulomb forces on the Position of the Pole in the Scattering Amplitude and on Its Residue” published in Phys.At.Nuclei, **73** (2010) 757

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Abstract:

Certain comments on the paper of Yu.V. Orlov, B. F. Irgaziev and L. I. Nikitina published in Phys.At.Nuclei, **73** (2010) 757 are made.

In [1], the explicit form of the nuclear vertex constant (NVC) for the virtual decay $a \rightarrow b+c$ with two charged particles (b and c) and an arbitrary orbital momentum l_a is derived for the standard effective-range expansion $K_{l_a}(k^2)$. There, as an example, the bound and resonance states of the lightest nuclei, including the ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei in the p -wave, are studied. It should be noted that the similar expressions have earlier been derived in [2–4] independently, but in the other forms. For example, two the similar expressions are derived in [4] for the asymptotic normalization coefficient (ANC), which is proportional to the nuclear vertex constant up to the known multiplicative factor [5]. One of them is valid for the neutral case and another one is valid only for the charged case, and the latter has no limit when a charge of a particle (either b or c) tends to zero.

As it is seen from here, at present obtaining a correct relation between the NVC (or ANC) and the parameters of the effective range expansion is of great interest since it makes it possible to determine the aforesaid fundamental characteristic bound state of the nucleus a in the $(b+c)$ -configuration and the parameters of the effective range expansion for the bc -scattering by selfconsistent way.

In work [1], the results of work [3] is also criticized. In particular, in Introduction of [1] the authors assert that “...a serious error of fundamental importance was made in [3]: Eq. (25)¹ in [3] (the numbering of the formulas in this and in the next section corresponds strictly to the numbering in [3]), which relates the binding energy to the “scattering length” and to the “effective range”, was written without allowance for Coulomb interaction (!). Equation (25), which is inappropriate in the case of charged particles, was used there to derive a formula for the elastic-scattering amplitude {see Eq.(23) in [3]}, ... The “hybrid” scattering amplitude obtained in this way (*determined by the expressions (23)–(25)*) does not have a pole at the binding energy ε_{bc} not allowing for Coulomb interactions or at the correct binding-energy value ε_{bc}^{NC} , which includes the Coulomb interaction.... Thus, expressions (30) and (31) in [3] for the vertex constant G_i^{NC} ($G_{bc;l_a s_a}$ in the denotation of [3]) characterizing the virtual decay of nucleus a to two charged fragments, $a \rightarrow d+c$, are erroneous.” (Here and below the numbering

¹In [3], there is the misprint in the expression (25) (see the works [6, 7]); further in [3], the correct expression was in reality used (here and below the italics are made by us).

of the formulas corresponds to [3], if another is not noted specially, and the phrases inside the quotation marks belong to authors of [1]).

We may agree with these asserts partially, namely, that is related to the expression (25). Nevertheless, in [3] a use of the approximated equation (25) (the connection equation for the bound $(b+c)$ state) in the partial scattering amplitude (23) does not influence the derivation of the expressions (30) and (31) since this equation modifies only the first term of the denominator of the amplitude (23) (or (24)), which does not depend in reality on the variable of the relative momentum k (or the energy E). Consequently, the result of differentiation of this denominator over the variable of E (or k) does not depend on a choice of the form of the equation (25). Unfortunately, the authors of [1] do not pay attention to this obvious fact. Therefore, in the chosen normalization for the Coulomb-nuclear part of the partial amplitude of bc -scattering given by the expression (19) (or (23)), the expressions (30) and (31) connecting NVC $G_{bc;l_a s_a}$ with the effective radius parameter are correct. One notes that these expressions were derived by us in two independent ways. Unfortunately, one cannot compare the expressions (30) and (31) of [3] with the analogous one (27) of [1] since a result of differentiation of the denominator over the variable k (or E) is not presented in [1]. In addition, the formula (27) derived in [1] can be obtained directly from the analogous one of [2] but derived earlier for the ANC (see p.350 there), if one makes use of the known relation between the ANC and NVC [5]. Therefore, in reality, the formula (27) of [1] was firstly obtained in [2], but not in [1], as it is asserted by authors of [1].

Besides, it should be noted that the normalization for the Coulomb-nuclear part of the partial amplitude (19) (or (23)) chosen in [3] differs from that in [1, 2, 4] by a factor of the Coulomb phase multiplicative $e^{2i\sigma_l(k)}$. Allowance of this factor in the corresponding expressions of [3] results in the renormalization of the right hand side of the expressions (31) and (34). In this case, the factor $K(\eta_B)$ entering in the nominator of the right hand side of the aforesaid expressions must be replaced by the factor $\Gamma^2(l_B + 1 + \eta_B)/(l_B!)^2 D_{l_B}(-i\eta_B)$.

It should be noted that in [7] the equation (25) of [3] has already been generated for charged particles (b and c), which transforms to the equation (25) when a charge of the particle b (or c) tends to zero. Combining of the expressions (30) and (31) with the generated equation (25) presented in [7] makes it possible to express the NVC (or ANC) through the parameter of the “scattering length” and, distinction on the similar relations (14), (17) and (18) of [4], this relation is valid both for the charged case and for the neutral one. Therefore, this combined expression can be applied for getting an information about the scattering data if a value of the NVC (or ANC) is known.

However, for some of the specific scattering considered in [8], including the αt -scattering too, the additional phase analysis performed by us, where the information about the “experimental” value for the corresponding ANC and the aforesaid generated equation [7] are taken into account, shows that in [3] a use of the approximation for the effective range expansion restricting by terms up to k^2 does not allow one to reproduce the corresponding shift-phase scattering at low energies by the selfconsistent manner. In [7], the new results for the modified values of the parameters of the effective-range expansion (“scattering length” and the “effective range”) and the p -wave phase shifts obtained for the αt -scattering have already been given.

In this connection, at present the aforesaid expression for the NVC (or ANC) and the

corresponding connection equation for the bound ($b+c$) state have been generated by us for the effective expansion function $K_{l_a}(k^2)$ restricting by terms up to k^6 . These expressions were also applied for an analysis of the experimental phase shift scattering considered in [8]. In particular, combining of these expressions with the known “experimental” values of the ANCs for the ground and first excited states of ${}^7\text{Li}$ in the $(\alpha+t)$ -channel [3] makes it possible one to reduce the number of the free effective expansion parameters on two. As a result, the values of these parameters found by this way reproduce rather well the experimental p - wave phase shifts for αt -scattering at energies up to about 5 MeV. This result and the similar ones for the other scattering considered in [8] will be presented for publication in a form of a separate paper.

It should be noted that these expressions can also be used for resonant states of the nucleus a . For this, the binding energy ε_{bc} (or ε_{bc}^{NC}) should be replaced by $-E^{(r)}+i\Gamma/2$, where $E^{(r)}(\Gamma)$ is the energy (width) of the resonant state of a .

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